The series in (11) is not necessarily asymptotic. For example, if $T_R = 1$, $\lambda = r^\nu$, $\nu = (4k + 2)/(2k + 3)$, and k is an integer, then $q_R = \sqrt{\gamma}K_n'(\sqrt{\gamma}R)/K_n(\sqrt{\gamma}R)$, where the index n = $(2k + 1)/2$. The MacDonald functions represent finite series by powers of $\gamma^{-1/2}$ and the ratio of polynomials is expanded in an analogous series with a finite radius of convergence.

## NOTATION

T, temperature; $T_R$, temperature at rim of opening; T*, approximate value of temperature; $q_R$, radial temperature gradient at edge of opening; $q_R$*, approximate value of gradient; $\gamma$, heat-transfer coefficient; r, $\varphi$, polar coordinates; R, radius of opening; $a_n$, $b_n$, functions of radius; $\lambda$, thermal conductivity; $M_n$, $L_n$, linear operators; $K_n$, MacDonald functions.

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# PHASE-TRANSITION KINETICS AND KINETIC EQUATIONS

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Kinetic equations for gas—liquid droplet and liquid—gas bubbles systems are derived and studied by the characteristic and moment methods.

In investigating the kinetics of phase transitions in multiphase media it is necessary to study formation of phase nuclei, together with their growth and decay. The kinetics of solid-phase nucleus formation in liquid condensation and gas-phase nucleus formation in boiling are described by the well-known equations of Vollmer, Becker and Deering, Frenkel' and Zel'dovich, Courtney, Probstein, Kantrowitz, et al. [1]. Further growth of these nuclei may be considered on the basis of growth or decay kinetics of unit nuclei for monodispersed media in the absence of nucleus interaction; however, for a large number of such nuclei this approach must be replaced by a kinetic description. A number of studies have presented various kinetic equations for particle distributions over velocity and dimensions for the processes of vapor condensation, liquid or vapor crystallization [1, 2], and sublimation, boiling, and cavitation processes, but these studies usually consider distribution functions over size alone [5], or over velocity without consideration of size [6], or with consideration of size, but without distribution over velocity [1, 3].

We will present below a generalized kinetic equation for the particle distribution function $f_i$ over time, coordinates, velocities, and particle size:

$$f_i(t,\ x,\ y,\ z,\ u,\ v,\ w,\ r). \tag{1}$$

Change in the distribution functions will be described by kinetic equations which have the form

$$\frac{\partial f_i}{\partial t} + \vec{u}_i \frac{\partial f_i}{\partial x_i} + \vec{F}_i \frac{\partial f_i}{\partial \vec{u}_i} + \frac{\partial \dot{r} f_i}{\partial r} = \sum_{i,j} J_{stij}. \tag{2}$$

For viscous liquid nuclei we may equate $\vec{F}_i$ to the Stokes friction force $\dot{u}_i = \alpha u_i$ or to other well-known expressions, for example, $\dot{u} = \alpha_1 u^2$, etc. Further, we assume

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$$\dot{u} = \varphi_0(u). \tag{3}$$

If we consider the free molecular mode of nucleus growth (decay), then the growth rate is defined by the well-known Knudsen formula

$$\dot{r} = \frac{\alpha_k}{\rho_b} \frac{p - p_\infty(T_s)}{\sqrt{2\pi RT}}, \tag{4}$$

which is independent of particle size r, or by other well-known formulas, for example, those of Maxwell or Fuchs for a continuous nucleus growth regime, as used in continual theory of dispersed systems.

In order to be able to consider the possibility of particle coaggulation or fractionation upon intercollision, the collision integral may be expressed in the following form [4]:

$$J_{st} = \frac{1}{2} \int\limits_0^v \beta(v - v_1, v_1) f(v - v_1, t) f(v_1, t) \, dv_1 -$$

$$- f(v, t) \int\limits_0^\infty \beta(v, v_1) f(v_1, t) \, dv_1 + \int\limits_v^\infty \gamma(v, v_1) f(v_1, t) \, dv_1 - \frac{f(v, t)}{v} \int\limits_0^v v_1 \gamma(v_1, v) \, dv_1. \tag{5}$$

In certain cases for the collision integral one may take

$$J_{st} = \frac{f_i - f_{i0}}{\tau_\Gamma}, \tag{6}$$

and in formation of monodispersed particles

$$J_{st} = I\delta(r - r_{cr}). \tag{7}$$

We will consider Eq. (2) in the one-dimensional case, using Eq. (7) as the collision integral. This equation may be solved by the characteristic method, as was done previously by one of the present authors for a plasma kinetic equation [7-9]. The operator I may be functional or integral.

The characteristic equations have the form

$$dt = \frac{dx}{u} = \frac{du}{\varphi_0(u)} = \frac{dr}{\dot{r}} = \frac{df}{I\delta(r - r_{cr}) - \frac{\partial \dot{r}}{\partial r} f}. \tag{8}$$

Considering that in a free molecular regime $\partial\dot{r}/\partial r = 0$, one can find a series of first integrals and use them to construct a general integral and find the explicit form of the distribution functions. Given an initially steady-state case, one can simply consider t as a parameter.

From the relationship dx/u = du/$\varphi_0$(u) we find the first integral

$$x = \int \frac{u \, du}{\varphi_0(u)} + C_1. \tag{9}$$

Let $\varphi_0(u) = au$; then x = (u/$a$) + $C_1$, and for $\varphi_0(u) = au^2$ we have x = (1/$a$)ln u + $C_1$. In the stationary case we may write $\dot{r}$ = uz'(x); then dx/u = dr/uz'(x) gives the following integral:

$$z(x) + C_2 = r, \tag{10}$$

where

$$z(x) = \frac{\alpha_k}{\rho_b} \int\limits_{x_1}^x \frac{p - p_\infty(T_s)}{u\sqrt{2\pi RT}} dx.$$

Finally, from the relationships dx/u = df/[I(x)$\delta$(r − $r_{cr}$)] we obtain yet another integral:

$$f = \int\limits_{x_1}^x \frac{dx I}{u} \delta(r - r_{cr}) = \int\limits_{x_1}^x \frac{I(\xi)}{u(\xi)} \delta[z(\xi) + C_2 - r_{cr}(\xi)] \, d\xi + C_3. \tag{11}$$

If we use the boundary conditions for $x = x_0$, $u = u_0$, $r = r_0$

$$f = f_0(x_0,\ u_0,\ r_0),$$

by the form of the function $f_0$ we can determine the form of the distribution function $f(x, u, r)$.

In the nonstationary case the problem is somewhat more complex, since not all the integrals of Eq. (8) can be found in explicit form. As before, Eq. (9) is valid. The relationship $dt = dx/u$ must be considered as a definition of $u$; however, the integral of the system $dr = \dot{r}(dx/u)$ cannot be found without knowledge of the concrete form of $\dot{r}$; i.e., we have

$$r = \int \dot{r}\frac{dx}{u} + C_2 ; \tag{12}$$

therefore, Eq. (11) takes on the more general form

$$f = \int\limits_{x_1}^{x} \frac{I(\xi,\ t)}{u(\xi,\ t)}\ \delta\left[\int\limits_{x_1}^{x} \dot{r}\frac{dx}{u} + C_2 - r_{\mathrm{cr}}(\xi,\ t)\right]d\xi + C_3. \tag{13}$$

The construction of the distribution function in explicit form becomes correspondingly more complicated.

The generalization to construction of distribution functions for particles of various types is obvious and can be performed by conventional methods [10].

The distribution functions may be constructed through use of various asymptotic methods, such as that of Chapman and Enskog, the moment method, etc. [7].

We define the n-th order moment of a distribution function as the quantity

$$M^{(n)} = \int\limits_{r_{\mathrm{cr}}}^{\infty} r^n f\, dr. \tag{14}$$

Multiplying Eq. (2) successively by $r^n$ and taking n moments of the kinetic equations, we obtain macroscopic characteristics averaged over size, which are important for construction of models of polydispersed media. We will apply the equation obtained to the kinetics of vapor bubble formation and growth.

The simplest model for change in bubble radius is based on Rayleigh's equation [11]

$$r\frac{d^2r}{dt^2} + \frac{3}{2}\left(\frac{dr}{dt}\right)^2 = \frac{p_0}{\rho}, \tag{15}$$

the integral of which for $p_0 = \mathrm{const}$ gives

$$\dot{r} = \sqrt{\frac{2}{3}\frac{p_0}{\rho r^3}(R_0^3 - r^3)}. \tag{16}$$

If we consider the forces of gas pressure in the bubble, then the pulsation equation

$$r\ddot{r} - \frac{3}{2}\dot{r}^2 = \frac{p_0}{\rho_l}\left(\frac{R_0}{r}\right)^{3\gamma} \tag{17}$$

allows determination of the rate of change of radius

$$\dot{r} = \sqrt{\frac{2}{3}\frac{p_0}{\rho_0(\gamma - 1)}\left[\left(\frac{R_0}{r}\right)^{3\gamma} - \left(\frac{R}{r}\right)^{3\gamma}\right]}. \tag{18}$$

Consideration of surface tension leads to the expression

$$r\ddot{r} + \frac{3}{2}\dot{r}^2 - \frac{1}{\rho}\left(p_\upsilon - p_\infty + p_0\frac{R_0^{3\gamma}}{r^{3\gamma}} - \frac{2\sigma}{r}\right) = 0, \tag{19}$$

and also gives the radius-dependent growth rate

$$\dot{r} = \left\{\frac{R_0^3}{r^3}\left[\dot{R}_0^2 - \frac{2}{3}\frac{1}{\rho}\left(p_\upsilon - p_\infty - \frac{3\gamma - 2}{\gamma - 1}p_0 + \frac{3\sigma}{R_0}\right)\right] + \frac{2}{3}\frac{1}{\rho}(p_\upsilon - p_\infty) - \frac{2\sigma}{\rho}\frac{1}{r}\right\}^{1/2}. \tag{20}$$

Dynamic bubble growth with consideration of liquid viscosity is described by the Rayleigh—Plesset equation

$$r\ddot{r} + \frac{3}{2}\dot{r}^2 = \frac{1}{\rho}\left(p_v - p_\infty + p_0\frac{R_0^{3\gamma}}{r^{3\gamma}} - \frac{2\sigma}{r} - 4\mu\frac{\dot{r}}{r}\right). \tag{21}$$

For all these equations $\partial r / \partial r = 0$. Assuming that on the bubble there act resistive forces proportional to velocity $u = \varphi_0(u)$, with consideration of the relationships presented here it is simple to integrate Eq. (8).

## NOTATION

$f_i$, distribution function; $t$, time; $x$, $y$, $z$, coordinates; $u$, $v$, $w$, velocities; $r$, particle size; $J_{st}$, elastic and inelastic collision integral; $I$, number of nuclei formed per unit time; $\delta$, Dirac function; $r_{cr}$, critical nucleus size; $\beta(v, v_1)$, probability of fusion of particles with volumes $v$ and $v_1$ upon collision; $\gamma(v, v_1)$, probability of formation of particles with volume $v$ upon fractionation of particles with volume $v_1$; $\tau_r$, relaxation time; $\dot{u}$, particle acceleration; $\dot{r}$, particle growth rate; $a_i$, coefficients; $T$, temperature; $p$, pressure; $\alpha_k$, accommodation coefficients; $\rho_b$, particle (droplet) density; $T_0$, saturation temperature; $p_\infty$, pressure at droplet boundary; $\varphi_0(u)$, specified law; $C_i$, integration constants; $M^{(n)}$, moments of distribution function; $r$, bubble radius; $\gamma$, adiabatic index; $\rho_l$, liquid density; $R_0$, initial bubble radius; $\sigma$, surface tension; $p_v$, vapor pressure within bubble; $p_\infty$, pressure far from bubble; $p_0$, initial vapor pressure within bubble.

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